"What comes beyond the standard Model", July 7, 2019
Bled Workshop

Emergent Gravity (from hidden sectors)

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Bibliography

Ongoing work with:

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Based on earlier work:

- with E. Kiritsis ArXiv:1408.3541

Introduction

• All fundamental theories of physics seem to rely on local symmetries.

• The list includes gauge theories and gravity.

• "Physics" = IR physics: this is the one we can test.

• There is an obvious big WHY?

• A hint about gauge theories may come from the "weak coupling intuition": without non-trivial vectors with gauge symmetries we cannot construct UV complete theories.

• There are many holes in this hint: the UV can be strongly coupled, the UV may be not a QFT, etc.

• H. Nielsen argued that gauge theories are generic low energy "artifacts" of non-gauge high-energy QFTs.

• It is not known whether such a statement can be true in general.

Gravity

• The other known interaction (gravity) has also a local invariance, but of quite a different form.

• The modern theory was born by trying to establish a theory of general covariance.

• The underlying invariance (diff invariance) however is not unique to gravity.

• Many QFTs can be diff. invariant without a fundamental graviton field. A common example is Nambu-Goto theory, a non-linear theory of scalars.

• It was Einstein's insight that by connecting the metric to diff invariance he linked general covariance and gravity.

• Unlike gauge theories, gravity as an IR theory is non-renormalizable.

• After efforts that span more than 50 years, the only examples of a quantized gravitational theory (with a semiclassical limit) are those included in the framework of string theory.

• This type of quantization of gravity is at the cost of including an "infinite number" of additional degrees of freedom and a new scale below the Planck scale: the string scale.

• Many people believe that (perturbative) string theory provides a UV complete theory of quantum gravity.

• This is false: (perturbative) string theory provides a (consistent) cutoff theory of quantum gravity.

• The cutoff is the Planck scale: string theory cannot answer questions involving (transfers of) energy at or beyond the Planck scale.

Attempts at quantizing gravity

• Many attempts in the past tried to capitalize on a winning strategy: resolving non-renormalizable interactions.

• This has worked several times. Famous and very distinct examples are the weak interactions and the strong interactions.

• The low-energy theory of the strong interactions is the IR-free (but non-renormalizable) theory of pions, that reminds quite well the problems with quantizing gravity.

• In that theory, it was eventually understood, that one can quantize the low energy degrees of freedom (pions), but this description has a cutoff, $\Lambda \sim GeV$.

• Instead, the high-energy degrees of freedom (quarks) are different and the QFT associated to them is UV complete (and effectively strongly-coupled in the IR)

• By analogy, this would suggest that the non-renormalizability of the graviton appears because of its compositeness: the graviton is a low-energy bound-state.*

• Many attempts were made to construct gravity theories where the graviton is a composite field, made out of more elementary fields, of all types: scalars, fermions, vectors etc.

• All such attempts failed to go beyond the classical and provide a dynamical explanation of why the bound state appears "feature-less" at low energies.

*This is not always true as we shall see.

• To contrast: In low-energy string effective actions (generalized gravity) the cutoff scale is the string scale.

• If the string dynamics is treated fully (and not only the low energy modes), then the cutoff becomes the Planck scale.

• In the string description there seems to be no hint of compositeness for the graviton.

The Weinberg-Witten nail

• The original version of the WW theorem assumes Lorentz invariance and a Lorentz covariant Energy-Momentum tensor.

• It proceeds to prove that no massless particle with spin $S > 1$ can couple to the stress tensor and no particles with $S > 1/2$ to a global conserved current.

• This does not rule out a theory that contains a "fundamental" massless graviton, as there exists a loop-hole: Conservation of the stress tensor makes it non-covariant, and projecting on helicity-2 is also non-covariant.

• There are also other ways of avoiding the theorem:

• In the case of massless vectors the statement says that no massless vector bound-states can couple to a conserved Lorentz-covariant global current.

• This is avoided in standard non-abelian gauge theories as the conserved current is not Lorentz-covariant (only up to a gauge transformation).

• There are more interesting counter-examples:

$*$ At the lower end of the conformal window in N=1 sQCD: the ρ-mesons become massless but also develop a gauge invariance at the same time.

Komargodski

• These are the "magnetic" gauge bosons of Seiberg.

• Their effective theory is renormalizable (being a standard non-abelian gauge theory).

• A final caveat: Lorentz invariance is crucial: otherwise the notion of masslessness is not well-defined. (even in dS or AdS the notion changes)

• In conclusion: WW can be evaded but it is a serious litmus test for all emergent graviton theories.

• We shall find out that although the essence of the WW theorem remains true, the effective theory for the massless graviton is none-of the massive gravity theories discussed, with all of their problems.

• Instead we shall find a fully covariant gravity theory where the mass of the graviton is due to the "(gravitational) Higgs effect".

The energy momentum tensor

• One would generically expect that the state generated out of the vacuum by the (conserved) energy-momentum tensor has the quantum numbers of the graviton

$$T_{\mu\nu}(p)|0\rangle \equiv |\epsilon_{\mu\nu}, p\rangle$$

• It is transverse because of energy conservation and can be made traceless.

• In weakly-coupled theories, this is a multi-particle state and therefore its interactions are expected to be non-local.

• If however, the interactions are strong and make this state a true tightly-bound state with a "size" L, then maybe we can reproduce gravity at scales $\gg L$.

• In particular, in the limit of infinitely-strong interactions we would expect to obtain a good point-like interaction theory for this bound-state graviton.

• Of course, WW constrains such bound-states but we will come later to such constraints, as they can be subtle.

• We must remember however that, in the presence of strong attractive interactions in the spin-two channel, there will be, generically speaking, a tower of states generated from the $T_{\mu\nu}$ acting on $|0\rangle$.

• If the theory is not conformal, such a spectrum will be discrete, and would associated with the (generically complex) poles of the two-point function of the stress tensor.

• Again, generically speaking, many such states will be unstable.

• If the theory is conformal, such states will form a continuum.

• This is the case in AdS/CFT.

The AdS/CFT paradigm

• AdS/CFT relates QFT to string theory and therefore to a theory of "quantum gravity"

• That a gauge theory at large-N can be described by a weakly coupled string theory was anticipated since the work of 't Hooft.

• There were twosurprises however:

♠ The string theory lives in higher than 4 dimensions.

♠ The associated effective theory is a theory of (generalized) gravity.

• Emergent dimensions are the avatar of the large N limit. Eigenvalue distributions become continuous extra dimensions as it was already seen in simpler matrix models.

• It is still a puzzle however, why the higher-dimensional theory has diffeomorphism invariance.

• This is seen acutely in attempts to reconstruct the bulk (gravity) theory from the QFT.

S. S. Lee and collaborators

• The masslessness of the higher-dimensional graviton was a related surprise: as we understand it now, it is related to energy conservation of the dual QFT.

Kiritsis, Adams+Aharony+Karch

• The holographic duality essentially implements what we discussed already: the graviton (and all other bulk fields) are composites of (generalized) gluons.

• Strong coupling in the QFT, as expected, was important in making the gravitational theory local (by suppressing string corrections)

• The other important ingredient is the large-N limit. It makes bulk fields (composites) interact weakly (despite the fact that the constituents interact strongly)

• It is important to understand that strongly-interacting gravitons have $M_s \simeq M_P$ and therefore their effective theory is badly non-local and nowhere near the semiclassical gravity we like.

We have learned that:

♠ Strong coupling in QFT makes gravitons tightly bound states.

♠ Large N makes gravitons weakly interacting.

and both of the above give an effective semiclassical theory of (composite) quantum gravity.

• AdS/CFT is a conjectured duality at all λ, N and claims that the QFT and the string theory are exactly dual.

• We believe that the duality can be used to define string theory and gravity non-perturbatively, by using the QFT to define the physics beyond the obvious cutoff of the string theory.

• This however, needs to be understood much better and it is a very difficult question, as in many cases it requires controlling physics beyond the perturbative.

WW versus AdS/CFT

- Is AdS/CFT compatible with the WW theorem?

- The WW theorem involves a subtle limit to define the helicity amplitudes that determine the couplings of massless states to the stress tensor or a local current.

- This limiting procedure is not valid in theories where the states form a continuum.

- This is the case in AdS/CFT.

- From the point of view of the QFT, the effective gravitational coupling is non-local.

- Therefore the WW-theorem does not apply to this case.

- What about not CFTs?

WW versus nAdS/nCFT

• Consider a familiar example: four-dimensional, large-N YM theory.

• Its string-theory dual is stringy near the AdS-boundary (weak QFT coupling).

• We expect a gravitational description at low energies (strong QFT coupling).

• The theory has a gap and a discrete spectrum and therefore the emergent gravitational interactions must be local

• Also gravity must be weakly coupled (and it is due to large N).

• The low energy spectrum contains two stable (lightest) massive scalars, and a massive graviton.

• A massive graviton is compatible with WW.

• It is also compatible with a fully diff invariant theory of a massless graviton in 5 dimensions.

• The 4d graviton mass is due to the non-trivial 5d background, hence a "Higgs effect".

• The above gives some credence to the idea that heavy-ion collisions form (unstable) black holes of a massive gravity theory that quickly Hawking evaporate.

The stress tensor "state" as a (classical) dynamical metric

• We would like to implement directly the idea of an emergent graviton as the state generated by the energy-momentum tensor.

• As a warm-up, we consider a translationally invariant QFT at a fixed background metric $\mathbf{g}_{\mu\nu}$ and a scalar source J coupled to a scalar operator O (for purposes of illustration).

• The presence of an arbitrary background metric $\mathbf{g}_{\mu\nu}(x)$ breaks translation invariance.

• A redefinition of the derivative→ covariant derivative "restores" energy-momentum conservation (in the absence of other non-constant sources):

$$T_{\mu\nu} \equiv \frac{1}{\sqrt{\mathbf{g}}} \frac{\delta S(\mathbf{g}, J)}{\delta \mathbf{g}^{\mu\nu}} \quad , \quad \nabla^{\mu}_{\mathbf{g}} \langle T_{\mu\nu} \rangle = 0$$

where $S(\mathbf{g}, J)$ is the action of the theory coupled to the fixed metric $\mathbf{g}$ and to the scalar source J.

• Consider the Schwinger functional

$$e^{-W(g_{\mu\nu},J)} = \int \mathcal{D}\phi \; e^{-S(\phi,g_{\mu\nu},J)}$$

• $g_{\mu\nu}$ is an arbitrary background metric, ϕ are the "quantum fields.

• We assume the presence of a cutoff that preserves diff invariance so that the quantities above are finite.

• This is tricky business but for the moment we can have dim-reg in mind.

• Sometimes $W(g, J)$ is unique (modulo renormalization) at the linearized level, sometimes it is not (improvement).

• Moreover there are ambiguities at the non-linear level.

• One can add diff-invariant functionals of the curvature for example.

• These correspond to "improvements" (ie alternative definitions of the stress tensor), both at the linear as also the non-linear level.

• We will call all of this "the scheme dependence" of the Schwinger functional.

• $W(g, J)$ is now diff-invariant:

$$W(g'_{\mu\nu}(x'), J(x')) = W(g_{\mu\nu}(x), J(x)) \quad , \quad g'_{\mu\nu} = g_{\rho\sigma}\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}$$

• W encodes the connected energy-momentum tensor and scalar correlations of the the original theory with metric $\mathbf{g}$.

$$\frac{1}{\sqrt{g}}\frac{\delta W(g)}{\delta g^{\mu\nu}}\Big|_{\substack{g=\mathbf{g}\\ J=0}} = \langle T_{\mu\nu}\rangle \quad , \quad \frac{\delta W(g)}{\delta J}\Big|_{\substack{g=\mathbf{g}\\ J=0}} = \langle O\rangle$$

- Therefore W can be expanded in the sources as follows

$$W(g) = \int d^dx\sqrt{g}\,[(g^{\mu\nu}-\mathbf{g}^{\mu\nu})(x)\langle T_{\mu\nu}(x)\rangle + J(x)\langle O(x)\rangle] +$$

$$+\frac{1}{2!}\int d^dx_1\, d^dx_2\sqrt{g(x_1)}\sqrt{g(x_2)}\,(g^{\mu\nu}-\mathbf{g}^{\mu\nu})(x_1)(g^{\rho\sigma}-\mathbf{g}^{\rho\sigma})(x_2)\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle + \cdots$$

- The (quantum) vev of the stress tensor is:

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{\det g}}\frac{\delta W(g,J)}{\delta g^{\mu\nu}}$$

and we will use it to define the associated effective action:

$$\Gamma(h,J,\mathbf{g}) \equiv -W(g,J) + \int d^4x\sqrt{g}\; h_{\mu\nu}\;(g^{\mu\nu}-\mathbf{g}^{\mu\nu})$$

via a modified Legendre transform.

• Γ is the generating functional of 1-PI energy-momentum tensor correlators and is extremal,

$$\frac{\delta \Gamma(h_{\mu\nu},J)}{\delta h_{\mu\nu}}\Big|_{g=\mathbf{g}} = 0 \quad , \quad \Gamma(h^*_{\mu\nu},J)\Big|_{g=\mathbf{g}} = W(\mathbf{g},J)$$

• The description above in terms of the energy-momentum tensor "effective action" is a theory of (classical) dynamical gravity.

• The dynamical metric is the energy-momentum tensor vev.

• Other sources like J represent energy-momentum currying sources.

• This description is fully diff-invariant by construction.

• The interactions mediated by this graviton are essentially summarizing exchanges of the energy-momentum tensor as we had postulated.

• The emergent graviton propagator (by construction) is generated by the poles of the energy-momentum tensor of the original theory.

An explicit IR parametrization

• We assume that the theory has a uniform mass gap for simplicity.

• We will now parametrize the Schwinger functional W in an IR expansion below the mass gap as

$$W(g,J)=\int\sqrt{g}\left[-V(J)+M^2(J)R(g)-\frac{Z(J)}{2}(\partial J)^2+\mathcal{O}(\partial^4)\right]$$

• We calculate

$$h_{\mu\nu}=\frac{V}{2}g_{\mu\nu}+M^2G_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)M^2-\frac{1}{2}T_{\mu\nu}+\cdots$$

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}R\;g_{\mu\nu}\quad,\quad T_{\mu\nu}=Z(J)\left(\partial_\mu J\partial_\nu J-\frac{1}{2}g_{\mu\nu}(\partial J)^2\right)$$

• The $h_{\mu\nu}$ appears uniquely determined, but there is a initial+boundary condition dependence in this formula.

• Note that for arbitrary external source J, this energy-momentum tensor vev is not conserved.

$$\nabla_g{}^{\mu} h_{\mu\nu} = \frac{1}{2}\left[V(J)' - Z(J)\Box_g J - \frac{1}{2}Z'(J)(\partial J)^2 - (M(J)^2)' \; R\right]\partial_\nu J$$

• We may now solve $g_{\mu\nu}$ as a function of $h_{\mu\nu}$:

$$g_{\mu\nu} = \tilde{h}_{\mu\nu} - \delta\tilde{h}_{\mu\nu} \quad , \quad \tilde{h}_{\mu\nu} = \frac{2}{V} h_{\mu\nu}$$

$$\delta\tilde{h}_{\mu\nu} = \frac{2}{V}\left[M^2\tilde{G}_{\mu\nu} - (\tilde{\nabla}_\mu\tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M^2\right] - \frac{1}{V}\tilde{T}_{\mu\nu} + \cdots$$

• All the tensors above are written in terms of $\tilde{h}_{\mu\nu}$.

• $\tilde{h}_{\mu\nu}$ is dimensionless and plays the role of the emergent dynamical metric.

• We may rewrite it as an Einstein equation coupled to "matter"

$$M^2\ \tilde G_{\mu\nu} = \frac{V(J)}{2}\left(\tilde h_{\mu\nu} - \mathbf{g}_{\mu\nu}\right) + \frac{1}{2}\tilde T_{\mu\nu}(J) + (\tilde\nabla_\mu\tilde\nabla_\nu - \tilde h_{\mu\nu}\tilde\Box)M(J)^2 + \cdots$$

• The effective gravitational equation above is equivalent to $\frac{\delta\Gamma}{\delta h_{\mu\nu}} = 0$.

• The emergent graviton is massless.

• The background metric $\mathbf{g}_{\mu\nu}$ appears as an external source and contributes like a cosmological constant.

• Other sources act as sources of energy and momentum.

• This description is non-singular only if $V \neq 0$.

• This is also why the WW is evaded.

• If $V = 0$, then the gravitational theory is non-local.

Gravitons from hidden sectors

• We have presented so far a proof of principle, on how to describe an emergent graviton that summarizes interactions in a QFT mediated by an energy-momentum tensor exchange.

• We have seen how the full non-linear diff-invariance and dynamical gravity appear.

• It is, in a sense, this mechanism which is responsible for the emergence of gravity in the dual description of a holographic theory, with a few caveats:

♠ Instead of a single four-dimensional graviton we have an infinite number, one for each pole of the two-point correlator.

♠ They combine into a higher-dimensional graviton in the appropriate number of dimensions.

♠ There are no easy detailed calculations that show this in a general holographic QFT (A proof for AdS/CFT?).

• However, in the real world, the graviton that couples to the SM stress tensor appears to be an additional dynamical field.

• How can we describe this as an emergent degree of freedom?

♠ It can emerge in the standard way from a "hidden sector".

♠ The hidden sector will be coupled to the SM at some high scale.

♠ Only a few interactions will survive in the IR between the two theories as all interactions will be IR-irrelevant.

♠ This will match with the IR-freedom and non-renormalizability of gravity.

• If we want this graviton to be tightly bound and weakly coupled, then this hidden sector theory must be a large-N, strongly coupled (ie holographic) QFT.

• We are led therefore to couple a large-N theory to the SM in a UV complete fashion.

• There is a unique way to do this without a "messenger sector": Couple the unique gauge invariant relevant operator of the standard model (the Higgs mass operator) to a scalar operator of the hidden theory with $\Delta \leq 2$.

Quiros+Delgado

• There are various reasons to assume that the hidden theory will not have such a scalar operator.

• One therefore should postulate a massive messenger sector to couple the two theories together.

The linearized coupling

• We therefore consider a coupling between the "hidden theory" and the "visible theory" of the form

$$S_{int} = \int d^4x \left(\lambda \mathsf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x) + \lambda' \mathsf{T}(x) \widehat{\mathsf{T}}(x) \right)$$

at a high scale M. This is an irrelevant coupling with $\lambda \sim M^{-4}$.

• $\mathsf{T}_{\mu\nu}$ is the SM energy-momentum tensor, $\widehat{\mathsf{T}}_{\mu\nu}$ is the hidden one.

• We also define

$$\mathfrak{c} \equiv \frac{\lambda'}{\lambda} \quad , \quad \mathbf{T}_{\mu\nu} \equiv \mathsf{T}_{\mu\nu} + \mathfrak{c}\, \mathsf{T}\, \eta_{\mu\nu}$$

so that

$$S_{int} = \lambda \int d^4x\, \mathbf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x)$$

• Note that the expectation value of the hidden energy momentum tensor, acts as an external metric for the SM.

$$\lambda \int d^4x \;\; \mathbf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x) \quad \to \quad \int d^4x \;\; \mathbf{T}_{\mu\nu}(x) h^{\mu\nu}$$

- To proceed further we write for the Schwinger functional

$$e^{-W(\mathcal{J})} = \int [D\Phi][D\widehat{\Phi}]\, e^{-S_{vis}(\Phi,\mathcal{J})-S_{hid}(\widehat{\Phi})-\lambda\int d^4x\, \mathbf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x)}$$

$$= \int [D\Phi][D\widehat{\Phi}]\, e^{-S_{vis}(\Phi,\mathcal{J})-S_{hid}(\widehat{\Phi})}\Bigg[1 - \lambda\int d^4x\, \mathbf{T}_{\mu\nu}(x)\, \widehat{\mathsf{T}}^{\mu\nu}(x)$$

$$+\frac{1}{2}\lambda^2\int d^4x_1 d^4x_2\, \mathbf{T}_{\mu\nu}(x_1)\mathbf{T}_{\rho\sigma}(x_2)\widehat{\mathsf{T}}^{\mu\nu}(x_1)\widehat{\mathsf{T}}^{\rho\sigma}(x_2) + \mathcal{O}(\lambda^3)\Bigg]$$

We will also use

$$\langle\widehat{\mathsf{T}}_{\mu\nu}(x)\rangle^{(0)} = \widehat{\Lambda}\,\eta_{\mu\nu}$$

where $\widehat{\Lambda}$ is a dimension-4 constant and the connected correlator

$$\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(x_1-x_2) = \langle\widehat{\mathsf{T}}^{\mu\nu}(x_1)\,\widehat{\mathsf{T}}^{\rho\sigma}(x_2)\rangle^{(0)}_{hid}$$

• We obtain

$$e^{-W(\mathcal{J})} = e^{-W_{hid}^{(0)}} \int [D\Phi]\, e^{-S_{vis}(\Phi,\mathcal{J})} \Big[1 - \lambda \widehat{\Lambda} \int d^4x\, \mathbf{T}(x)$$

$$+ \frac{1}{2}\lambda^2 \widehat{\Lambda}^2 \int d^4x_1 d^4x_2\, \mathbf{T}(x_1)\mathbf{T}(x_2)$$

$$+ \frac{1}{2}\lambda^2 \int d^4x_1 d^4x_2\, \mathbf{T}_{\mu\nu}(x_1)\, \mathbf{T}_{\rho\sigma}(x_2) \widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(x_1 - x_2) + \mathcal{O}(\lambda^3) \Big]$$

• The coupling has introduced the following effective interactions in the visible theory:

$$\delta S_{vis} = \lambda \widehat{\Lambda} \int d^4x\, \mathbf{T}(x) - \frac{1}{2}\lambda^2 \int d^4x_1 d^4x_2\, \mathbf{T}_{\mu\nu}(x_1)\, \mathbf{T}_{\rho\sigma}(x_2)\, \widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(x_1 - x_2)$$

• The second term can be written in momentum space as

$$\delta S_{vis}^{TT} \equiv -\frac{1}{2}\frac{\lambda^2}{(2\pi)^4} \int d^4k\, \mathbf{T}_{\mu\nu}(-k)\, \mathbf{T}_{\rho\sigma}(k)\, \widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$$

and is an induced quadratic energy-momentum interaction in the visible theory.

• This interaction can be reformulated in terms of a classical spin-2 field $h_{\mu\nu}$

$$\delta S^{TT}_{eff} = \int d^4k \left[-h_{\mu\nu}(-k)\mathbf{T}^{\mu\nu}(k) + \frac{(2\pi)^4}{2\lambda^2} h_{\mu\nu}(-k) \;\; \mathcal{P}^{\mu\nu,\rho\sigma}(k) \;\; h_{\rho\sigma}(k) \right]$$

• The inverse propagator $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ of the emerging spin-2 field is the inverse of the hidden sector 2-point function $\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$.

• Integrating out $h_{\mu\nu}$ we obtain our original expression.

• It remains to examine under what circumstances $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ is well-defined and what tensor structures it involves.

• We assume that the hidden theory is a Lorentz-invariant QFT with a mass gap.

• Then the Ward identities associated with translations imply that the general form of the 2-point function $\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$ is

$$\widehat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k) = \widehat{\Lambda}\Big(-\eta^{\mu\nu}\eta^{\rho\sigma}+\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\rho\nu}\Big)+\widehat{\mathsf{b}}(k^2)\Pi^{\mu\nu\rho\sigma}(k)+\widehat{\mathsf{c}}(k^2)\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\pi^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \quad , \quad \Pi^{\mu\nu,\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k)$$

$$k_\mu \; \pi^{\mu\nu} = k_\mu \; \Pi^{\mu\nu,\rho\sigma} = 0$$

• The first term on the RHS arises from a contact term contribution to the Ward identity.

• The only combination of tensor structures which is analytic at quadratic order in momentum, in the long-wavelength limit $k^2 \to 0$, is the one that has

$$\widehat{\mathsf{b}}(k^2) = \widehat{\mathsf{b}}_0 k^2 + \mathcal{O}(k^4) \;, \quad \widehat{\mathsf{c}}(k^2) = -2\widehat{\mathsf{b}}_0 k^2 + \mathcal{O}(k^4)$$

• In that case, the low-momentum expansion of $\widehat{\mathsf{G}}^{\mu\nu\rho\sigma}(k)$ up to quadratic order in momentum is

$$\widehat{\mathsf{G}}^{\mu\nu\rho\sigma}(k) = \widehat{\Lambda}\Big(- \eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\Big)$$

$$+\widehat{\mathsf{b}}_0\Big[k^2\,(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - 2\eta^{\mu\nu}\eta^{\rho\sigma})$$

$$-\eta^{\mu\rho}k^\nu k^\sigma - \eta^{\nu\sigma}k^\mu k^\rho - \eta^{\mu\sigma}k^\nu k^\rho - \eta^{\nu\rho}k^\mu k^\sigma + 2\eta^{\mu\nu}k^\rho k^\sigma + 2\eta^{\rho\sigma}k^\mu k^\nu\Big] + \mathcal{O}(k^4)$$

• A term proportional to $\frac{k^\mu k^\nu k^\rho k^\sigma}{k^2}$ has cancelled out.

• As a simple check, we consider the two-point function in a theory of N^2 decoupled free massive bosons arranged as an $N \times N$ matrix $\widehat{\phi}$

$$S_{hidden} = -\frac{1}{2}\int d^4x\,\mathrm{Tr}\left(\partial_\mu\widehat{\phi}\,\partial^\mu\widehat{\phi} + m^2\widehat{\phi}^2\right)$$

• In dimensional regularization ($\varepsilon = 4 - d \to 0$) the two-point function can be evaluated explicitly by performing the requisite Wick contractions.

$$\widehat{\Lambda} = -\frac{N^2}{64\pi^2}\Gamma\left(\frac{\varepsilon}{2}\right) m^4$$

$$\widehat{\mathrm{b}}(k^2) = -\frac{N^2}{16\pi^2}\Gamma\left(\frac{\varepsilon}{2}\right)\left[\frac{m^2}{12}k^2 + \frac{1}{120}k^4\right]$$

$$\widehat{\mathrm{c}}(k^2) = -\frac{N^2}{16\pi^2}\Gamma\left(\frac{\varepsilon}{2}\right)\left[-\frac{m^2}{6}k^2 + \frac{1}{20}k^4\right] \ .$$

• If we use a hard momentum cutoff, Λ, we will obtain terms of order Λ^4 and $\Lambda^2 m^2$ for $\widehat{\Lambda}$ and terms of order Λ^2 for the other two coefficients.

• These violate the Ward identities and should be subtracted.

• The presence of $\widehat{\Lambda} \neq 0$ facilitates a well-defined inversion of $\widehat{\mathrm{G}}^{\mu\nu\rho\sigma}(k)$ in the long-wavelength limit $k^2 \to 0$.

• If $\widehat{\Lambda}=0$, the two-point function has zero modes which are proportional to k^μ and is therefore not invertible.

• In this case, one must invert in the space orthogonal to the zero modes. This gives rise to a non-local effective theory for the graviton.

• Up to quadratic order in the momentum expansion

$$\mathcal{P}^{\mu\nu\rho\sigma}(k) = -\frac{1}{4\widehat{\Lambda}}\left(\eta^{\mu\nu}\eta^{\rho\sigma}-\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho}\right)$$

$$+2\widehat{\mathrm{b}}_0\widehat{\Lambda}^{-2}\left[\frac{k^2}{8}\left(\eta^{\mu\nu}\eta^{\rho\sigma}-\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho}\right)\right.$$

$$\left.+\frac{1}{8}\left(\eta^{\nu\sigma}k^\mu k^\rho+\eta^{\nu\rho}k^\mu k^\sigma+\eta^{\mu\sigma}k^\nu k^\rho+\eta^{\mu\rho}k^\nu k^\sigma\right)\right]+\mathcal{O}(k^4)$$

• It would seem that this is the propagator of massive graviton, but this is an illusion.

Emergent quadratic gravity

- We now re-define:

$$h_{\mu\nu} = -\mathfrak{h}_{\mu\nu} + \frac{1}{2}\mathfrak{h}\,\eta_{\mu\nu} + \lambda\widehat{\Lambda}\,\eta_{\mu\nu}\ , \quad \mathfrak{h} = \mathfrak{h}^{\rho\sigma}\eta_{\rho\sigma}$$

$$\mathfrak{T}^{\mu\nu} \equiv \mathbf{T}^{\mu\nu} - \frac{(2\pi)^4}{\lambda}\left(1 + \frac{1}{2\lambda\,\widehat{\Lambda}}\right)\eta^{\mu\nu}\ , \quad \mathfrak{T} = \mathfrak{T}^{\mu\nu}\eta_{\mu\nu}$$

- The full effective action of the visible QFT at this order in the λ-expansion and at the two-derivative level is

$$S_{eff} = S_{vis} + \int d^4x\,\left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) + \frac{1}{16\pi G}\int d^4x\left[\sqrt{g}\,(R+\Lambda)\right]^{(2)}_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

with the identification of parameters

$$\Lambda = \frac{\widehat{\Lambda}}{\widehat{b}_0}\ , \quad \frac{1}{16\pi G} \equiv M_P^2 = -\frac{(2\pi)^8\ \widehat{b}_0}{\lambda^2\widehat{\Lambda}^2}$$

• The sign of Newton's constant is positive when $\widehat{b}_0$ is negative

• This seems to be the case with simple QFTs but we have no general proof.

• The sign of the cosmological constant is opposite ($\hat{b}_0 < 0$) to the sign of $\widehat{\Lambda}$.

• The second term, which describes the coupling of the visible QFT to the emergent graviton, can be expressed in terms of the original energy-momentum tensor of the visible QFT, $T_{\mu\nu}$,

$$\int d^4x \left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T} \right) = \int d^4x\, \mathfrak{h}_{\mu\nu} \left(T^{\mu\nu} + \frac{1}{\lambda}\left(1 + \frac{1}{2\lambda\,\widehat{\Lambda}}\right)\eta^{\mu\nu} \right)$$

• There is a non-trivial shift of the energy due to the coupling of the two theories.

Emerging quadratic gravity:Comments

• A coupling of stress tensors between two theories induces gravity at the quadratic level.

• This is true in the generic case: $\widehat{\Lambda} \neq 0$.

• Otherwise the graviton theory is non-local.

• There is always an effective cosmological constant for the emerging gravity in the local case.

• There is also a shift of the stress tensor giving a “dark” energy. It is a reflection of the coupling to the hidden theory.

• We parametrize $\lambda = (2\pi)^4 \; N^{-1} M^{-4}$ where M a large scale controlling the coupling of the two theories and N the number of colors of the hidden theory.

• Also from calculations

$$\hat{b}_0 = -\kappa N^2 m^2 \quad , \quad \kappa \sim O(1) \quad , \quad \hat{\Lambda} = \epsilon \; N^2 \; m^4 \quad , \quad \epsilon = \pm 1 \tag{1}$$

We may now calculate the relevant ratios of scales

$$\frac{\Lambda}{M_P^2} = -\frac{\epsilon}{\kappa^2 x^2} \quad , \quad \frac{\Lambda_{dark}}{M_P^2} = -\frac{\frac{N}{x} + \frac{\epsilon}{2(2\pi)^4}}{(1+4\mathfrak{c})\kappa^2 \; x^2} \tag{2}$$

$$\frac{\Lambda_{dark}}{\Lambda} = \frac{\epsilon\frac{N}{x} + \frac{1}{2(2\pi)^4}}{(1+4\mathfrak{c})} \quad , \quad \frac{M^4}{M_P^4} = \frac{1}{\kappa^2 x^3} \quad , \quad x \equiv \frac{M^4}{m^4} \gg 1 \tag{3}$$

• We always have semiclassical gravity, $\Lambda \ll M_P^2$.

• If $N \lesssim x$ then

$$\Lambda \quad \sim \quad \Lambda_{\text{dark}} \quad \sim \quad \mathcal{O}(m^2) \quad \ll \quad M^2 \quad \ll \quad M_P^2$$

• If $x \ll N \ll x^{\frac{3}{2}}$ then

$$\Lambda \quad \ll \quad \Lambda_{\text{dark}} \quad \ll \quad M^2 \quad \ll \quad M_P^2$$

• If $x^{\frac{3}{2}} \ll N \ll x^3$ then

$$\Lambda \quad \ll \quad M^2 \ll \quad \Lambda_{\text{dark}} \quad \ll \quad M_P^2$$

• If $N \gg x^3$ then

$$\Lambda \quad \ll \quad M^2 \quad \ll \quad M_P^2 \quad \ll \quad \Lambda_{\text{dark}}$$

• For phenomenological purposes $x \lesssim 10^{20}$ so that the messenger scale is above experimental thresholds.

• Note that so far the SM quantum effects are not included.

The non-linear analysis

- We start again from the Schwinger functional of the coupled QFTs

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi]\,[D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\mathbf{g},\hat{\mathcal{J}}\right)-S_{int}\left(\mathcal{O}^i,\widehat{\mathcal{O}}^i,\mathbf{g}\right)}$$

- Φ^i and $\widehat{\Phi}^i$ are respectively the (quantum) fields of the visible QFT and the hidden $\widehat{\text{QFT}}$.

- $\mathcal{J}$ and $\hat{\mathcal{J}}$ are (scalar) sources in the visible and hidden theories respectively.

- For energies $E \ll M$, we can integrate out the hidden theory and obtain

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi][D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\hat{\mathcal{J}},\mathbf{g}\right)-S_{int}}$$

$$= \int [D\Phi]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-\mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g},\right)}$$

- The interaction part is defined as:

$$S_{int} = \int d^4x\sqrt{\mathbf{g}}\sum_i \lambda_i\,\mathcal{O}_i(x)\,\widehat{\mathcal{O}}_i(x)$$

• The functional $\mathcal{W}\left(\mathcal{O}^i + \hat{\mathcal{J}}^i, \mathbf{g},\right)$ represents the generating functional for the hidden theory with the original fixed sources $\hat{\mathcal{J}}$ and $\mathbf{g}_{\mu\nu}$ and new dynamical sources $\mathcal{O}^i$ given by the operators of the visible theory.

• The low-energy interactions of the visible theory are now controlled by the following action

$$S_{total} = S_{visible}\left(\Phi, \mathcal{J}, \mathbf{g}\right) + \mathcal{W}\left(\mathcal{O}^i + \hat{\mathcal{J}}^i, \mathbf{g}\right)$$

• We now put the full theory on a curved manifold with metric $g_{\mu\nu}$ and define again the generating functional in the presence of the background metric as

$$e^{-W(\mathcal{J},g,\hat{\mathcal{J}})} = \int [D\Phi]\, e^{-S_{visible}(\Phi,\mathcal{J},g) - \mathcal{W}(\mathcal{O}^i + \hat{\mathcal{J}}^i, g)}$$

• We define

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{g}} \frac{\delta \mathcal{W}\left(\mathcal{O}^i, g, \hat{\mathcal{J}}\right)}{\delta g^{\mu\nu}}\Big|_{g_{\mu\nu} = \mathbf{g}_{\mu\nu}} = \langle \widehat{\mathsf{T}}_{\mu\nu} \rangle$$

• This will eventually play the role of an emergent metric for the visible theory.

• The diffeomorphism invariance of the functional $W(\mathcal{J}, g, \hat{\mathcal{J}})$ is reflecting (as usual) the translational invariance of the underlying QFT.

• We may now invert the previous equation to obtain:

$$g_{\mu\nu} = g_{\mu\nu}\left(\mathcal{O}^i + \hat{\mathcal{J}}^i, h_{\mu\nu}\right)$$

• It can be shown that to leading order in the derivative expansion $g_{\mu\nu} \sim h_{\mu\nu}$.

• We define the Legendre-transformed functional

$$S_{eff}(h, \Phi, \mathcal{J}, \hat{\mathcal{J}}, \mathbf{g}) = S_{vis}(\mathbf{g}, \Phi, \mathcal{J}) - \int d^4x \sqrt{g(\mathcal{O}^i + \hat{\mathcal{J}}^i, h)}\, h_{\mu\nu} \times$$
$$\times \left[g^{\mu\nu}(\mathcal{O}^i + \hat{\mathcal{J}}^i, h) - \mathbf{g}^{\mu\nu}\right] + \mathcal{W}\left(\mathcal{O}^i + \hat{\mathcal{J}}^i, g(\mathcal{O}^i + \hat{\mathcal{J}}^i, h)\right)$$

We can show that:

♠ This functional satisfies

$$\left.\frac{\delta S_{eff}}{\delta h_{\mu\nu}}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = 0$$

♠ These are the emerging non-linear gravitational equations.

♠ When evaluated in the solution of the above equation gives the original action.

$$, \quad S_{eff}\Big|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = S_{visible} + \mathcal{W}\left(\mathcal{O}^i + \hat{\mathcal{J}}^i, \mathbf{g}\right)$$

• Therefore, $S_{eff}(h, \Phi, \mathcal{J}, \hat{\mathcal{J}}, \mathbf{g})$ is the emergent gravity action that generalizes the linearized computation.

Comments and Open ends

- The separation of the interacting theories into two pieces is not unique. The respective gravities are related by metric redefinitions.

- The effective cosmological constant is (roughly) the sum of the two cosmological constants.

- IN all of this we assumed that the contribution of messenger physics to the cosmological constants is suppressed. This can be easily be case.

- The "dark energy" comes from the hidden theory.

- The emergent graviton is "massless" and the full diff-invariance intact.

- One point of view is that the WW Theorem is inapplicable because of the presence of a non-zero cosmological constant, and therefore a non-trivial gravitational background.

- An alternative point of view is that effectively the graviton is “massive” (because of the cosmological constant) but the mass comes from the “gravitational Higgs effect”.

- Additional sources in the hidden theory may provide new sources of ”dark” components: energy, matter etc.

- We can integrate-in many other fields. Most of them however will have masses of $\mathcal{O}(M) \sim M_P$. The only protected ones, are the graviton, the universal axion and global conserved currents (graviphotons).

- When the hidden theory is a holographic QFT then this description should transform into the brane-in-bulk (or brane-world) description.

- We can entertain the possibility of several hidden sectors. The graviton is a combination of the stress tensors with dominant contribution from the largest N.

- Is emergent gravity always attractive?

• The signature of the metric in emerging gravity can change. If a stress tensor is close to that of a cosmological constant then the signature is Minkowski. If it is of the photonic type, it has Euclidean signature.

• Therefore we conclude that if the hidden theory is near it ground state, it will be dominated by its cosmological constant and the signature is Minkowski. If the hidden theory is in a highly excited state, then the emerging metric will become of Euclidean signature.

• What is this all good for?

♠ It can provide useful intuition on how gravity can emerge

♠ It can provide credible models for cosmology, as the QFT formulation changes the notion of what is "natural" or "generic".

♠ It may lead to potential interesting models for dark energy and the cosmological constant as well as the hierarchy problem.

THANK YOU!

The Weinberg-Witten loop-hole

• In GR the stress tensor is not conserved but covariantly conserved.

• One can add corrections to the tress tensor (involving also the flat metric) to make is strictly conserved and Lorentz covariant. This is however NOT a tensor under general coordinate transformations (but this is OK with WW).

• To make a pure helicity-two state, we must project out the (unphysical) helicity 1 and 0 states. This projection is NOT Lorentz covariant (but only up to a gauge transformation).

• We may appeal to diff-invariance to decouple the helicity 0 and 1 states but then we are stuck: $T_{\mu\nu}$ is now NOT fully covariant.

• Therefore GR and many other theories with an explicit dynamical graviton avoid the WW theorem.

Translation Ward identity

• We consider a theory with Lagrangian $\mathcal{L}$. For concreteness, we focus on four-dimensional QFTs.

• Under an infinitesimal diffeomorphism generated by a vector ξ_μ

$$\delta_\xi \mathcal{L} = \frac{1}{2} (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu) T^{\mu\nu}$$

$$\delta_\xi T^{\mu\nu} = \xi^\sigma \partial_\sigma T^{\mu\nu} + T^{\sigma\nu} \partial^\mu \xi_\sigma + T^{\mu\sigma} \partial^\nu \xi_\sigma$$

• The invariance of the partition function $Z = e^{i \int d^4x \, \mathcal{L}}$ under the infinitesimal translation implies the conservation equation

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

• Similarly, the invariance of the one-point function of the energy-momentum tensor

$$\langle T^{\rho\sigma}(y)\rangle = \frac{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}\, T^{\rho\sigma}(y)}{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}}$$

under the infinitesimal translations implies the Ward identity

$$-i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle + \delta(x-y)\langle\partial^\nu T^{\rho\sigma}(x)\rangle + \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$-\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) - \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

• In addition, Lorentz invariance implies that the one-point function of the energy-momentum tensor is

$$\langle T^{\mu\nu}(x)\rangle = a\eta^{\mu\nu}$$

where a is a dimensionfull constant.

Consequently, we set

$$\langle\partial^\nu T^{\rho\sigma}(x)\rangle = 0$$

and use it to simplify the Ward identity

$$i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle - \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$+\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) + \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

• In momentum space we obtain instead:

$$k_\mu\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle = ia\left(-k^\nu\eta^{\rho\sigma} + k^\rho\eta^{\nu\sigma} + k^\sigma\eta^{\rho\nu}\right)$$

• This allows us to deduce the 2-point function as **??**

$$\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle$$

$$= ia\left(-\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) + b(k^2)\,\Pi^{\mu\nu\rho\sigma}(k) + c(k^2)\,\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\Pi^{\mu\nu\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k) \quad , \quad \pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

Aside: String theory vs the swampland

• Conjectures talk about "quantum gravity" but everyone means "string theory"

• The (plausible) assumption that string theory is the space of large-N strongly coupled QFTs, has an automatic avatar:

• The "swampland" corresponds to QFTs that are either weakly-coupled, or are not at large N.

• This explains for example, the generic towers of states that appear at the boundaries of moduli spaces.

• It also suggests why there might be no de Sitter solution in "string theory".

• The notion of string theory used above is certainly more general that the conventional one based on 2d CFTs

• It involves also 3, 4, 5 and 6-dimensional CFTs.

• It might be illuminating to try to see the swampland conjectures via this point of view.

Higher spin

• It is one of the obvious next questions to ask: what about doing this for other operators of your QFT:

• For fields up to $S = 1/2$ this is a standard procedure, and has been done in many contexts.

• The case of $S = 1$ is interesting as it would describe emergent gauge theory. It is qualitatively different than the gravity case.

• When $S > 2$ one can again do the same procedure as here.

• In that case however for interacting theories, higher spin fields are not conserved. The effective theory one obtains will be massive, with characteristic mass the overall cutoff (in string theory this is the string scale).

• They are therefore less interesting for low-energy physics.

• In a free QFT however they are conserved and then one can construct massless actions (of an infinite number of them)

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Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 2 minutes
- Gravity 4 minutes
- Attempts at quantizing gravity 7 minutes
- The Weinberg-Witten theorem 9 minutes
- The energy-momentum tensor 11 minutes
- The AdS/CFT paradigm 14 minutes
- WW versus AdS/CFT 15 minutes
- WW versus nAdS/nCFT 16 minutes
- The stress-tensor state as a (classical) dynamical graviton 23 minutes
- An explicit IR parametrization 28 minutes

- Gravitons from hidden sectors 29 minutes
- The linearized coupling 44 minutes
- Emerging quadratic gravity 48 minutes
- Emerging quadratic gravity:Comments 52 minutes
- The non-linear analysis 58 minutes
- Outlook 60 minutes

- Subtleties of WW 61 minutes
- Translation Ward Identities 62 minutes
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